



THOMAS' Twelfth Edition
CALCULUS
Early Transcendentals

THOMAS' CALCULUS EARLY TRANSCENDENTALS

Twelfth Edition

Based on the original work by

George B. Thomas, Jr.

Massachusetts Institute of Technology

as revised by

Maurice D. Weir

Naval Postgraduate School

Joel Hass

University of California, Davis

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